Clarifying Einstein's First Derivation for Mass-Energy Equivalence and Consequently Making Ives's Criticism a Void

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Abstract

We study physical situation considered by Einstein (Ann. Physik, 17, 1905) for his first derivation of mass-energy equivalence. Einstein introduced a constant C in his derivation and reasoning surrounding C and equations containing C caused criticism by Ives. Here we clarify Einstein's derivation and obtain a value for constant C. The obtained zero value for C suggests alternative explanation for Einstein's derivation and makes Ives's criticism a void and for which details are also presented in this paper.

I. INTRODUCTION

Different forms of mass-energy equivalence relation existed even before Einstein's first derivation of the relation[1] and which have been reviewed along with other developments on the relation after the year 1905 (see Ref.[2] and references cited therein). The focus here is on a century old Einstein's first derivation which has remained persistently debatable for its correctness and completeness[2,3] after the emergence of Ives' work[4] suggesting circular argument in the derivation. Here we show that Einstein's derivation contains hidden but valid condition. Under the presence of the condition we further obtain a value of constant C which Einstein invoked in his derivation (see Eqs. (4) and (5) below) leading to criticism by Ives. The obtained zero value for C in the present work makes Ives's criticism a void and is also shown in this paper. We first describe Einstein's derivation briefly along with our important notes written in *italics*, then present the analysis on hidden condition, value of C and analysis making Ives's criticism a void.

II. EINSTEIN'S DERIVATION

Consider a 'stationary' reference frame S_s with coordinate axes (x, y, z) and another reference frame S_v with axes (ξ, η, ζ) having constant translational velocity $\mathbf{v}$ as measured in S_s. Also, consider coordinate axes of S_v to be parallel to coordinate axes of S_s and origin of S_v in translational motion along the x axis of S_s with velocity magnitude $|\mathbf{v}| = v$. Consider a body of mass M_s at rest in S_s at some elevation and at some instance it emits in two opposite directions (along the x axis) equal quantity of light having energy $L/2$ where M_s and L are measured in S_s. The conservation of energy principle for this situation in S_s can be written as

$$E_0 = E_1 + \frac{L}{2} + \frac{L}{2} \tag{1}$$

where E_0 and E_1 are, respectively, total energy of the body before and after the emission of the light as measured in S_s.

It should be noted that Einstein did not consider any gravitational field in his derivation, otherwise gravitation effect on L should be included in Eq. (1). Which means that L should be replaced by $L(1 + \frac{\Phi}{c^2})$ where Φ is gravitational potential at the location of the body and c is speed of light[5]. So Einstein derivation of mass-energy equivalence is valid in the absence of

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gravitational field. In fact, Einstein derived gravitation of energy afterward in the year 1911. The derivation of mass-energy equivalence relation with inclusion of effect of gravitational field will be presented elsewhere.

The conservation of energy principle for the body as observed from S_v can be written as

$$H_0 = H_1 + \frac{L}{\sqrt{1 - v^2/c^2}} \tag{2}$$

where H_0 and H_1 are, respectively, total energy of the body before and after the emission of the light as measured in S_v. Subtracting Eq. (1) from Eq. (2) yields

$$(H_0 - E_0) - (H_1 - E_1) = L[\frac{1}{\sqrt{1 - v^2/c^2}} - 1]. \tag{3}$$

Einstein then provided following argument (hereafter, this argument is referred to as *EA*).

EA: "Thus it is clear the difference $H - E$ can differ from the kinetic energy K of the body, with respect to the other system (ξ, η, ζ), only by an additive constant C, which depends on the choice of the arbitrary additive constants of the energies H and E". And he wrote

$$H_0 - E_0 = K_0 + C, \tag{4}$$

$$H_1 - E_1 = K_1 + C. \tag{5}$$

Here K_0 and K_1 are, respectively, kinetic energy of the body before and after the emission of the light as measured in S_v.

It should be noted that these two Eqs. (4) and (5) with constant C written by Einstein have caused much confusion among researchers. The argument EA of Einstein and Eqs. (4) and (5) became cause for Planck's objection and criticism by Ives[4] suggesting flaw (as described in footnote[6]) in the Einstein's derivation. So if we do not invoke the argument and Eq. (4) and (5), Ives's criticism becomes void and this related analysis is presented in the next section.

Using Eqs. (4) and (5), Einstein obtained from Eq. (3)

$$K_0 - K_1 = L[\frac{1}{\sqrt{1 - v^2/c^2}} - 1] \tag{6}$$

Einstein then neglected fourth and higher orders terms in v in the expansion of right hand side of Eq. (6) and simplified Eq. (6) to

$$K_0 - K_1 = \frac{1}{2}\frac{L}{c^2}v^2 \tag{7}$$

and concluded "If a body gives off the energy L in the form of radiation, its mass diminishes by L/c^2". We should mention that Stachel and Torretti[3] showed that the approximation involved in Eq. (7) is not required to arrive at the conclusion when exact expressions for kinetic energies K_0 and K_1 are used in Eq. (6).

III. HIDDEN CONDITION, VALUE OF C AND VOIDING IVES'S CRITICISM

The correctness of Einstein's derivation depend on the correctness of Eqs. (4) and (5). Now we obtain hidden condition in Einstein's derivation under the assumption that Einstein's Eqs. (4) and (5) are correct. So if we find the obtained condition to be valid, that would suggest correctness of Eqs. (4) and (5) and Einstein's derivation. Then we show that Eq. (6) can be derived without using EA and Eqs. (4) and (5) thus voiding Ives's criticism which is based on EA and Eqs. (4) and (5).

Consider the body as a system in thermodynamic sense. In general the total energy of the body (system) is summation of gravitation potential energy P, kinetic energy K and internal energy. In the case of Einstein's derivation, as gravitation potential was not present as pointed out above, we write total energies (E_0, E_1, H_0 and H_1) before and after the emission in reference frames S_s and S_v in terms of internal and kinetic energies only.

In S_s, kinetic energy of the body is zero thus total energy of the body before and after the emission can be written as

$$E_0 = M_s I_s, \tag{8}$$

$$E_1 = (M_s - m_s)I_s'. \tag{9}$$

Here M_s is mass of the stationary body before the emission, m_s is decrease in mass of the body due to the emission, I_s and I_s' are internal energy per unit mass of the body before and after the emission, respectively, and all are measured in S_s.

As measured in S_v, we denote the mass of the moving body before the emission by M_v, decrease in mass of the body due to the emission by m_v, internal energy per unit mass of the body before and after the emission by I_v and I_v', respectively. With these notations we can write total energy of the body before and after the emission as measured in S_v as

$$H_0 = K_0 + M_v I_v, \tag{10}$$

$$H_1 = K_1 + (M_v - m_v)I_v'. \tag{11}$$

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Subtracting Eq. (8) from Eq. (10) and Eq. (9) from Eq. (11), we obtain

$$H_0 - E_0 = K_0 + \left[M_v I_v - M_s I_s \right], \tag{12}$$

and

$$H_1 - E_1 = K_1 + \left[(M_v - m_v)I'_v - (M_s - m_s)I'_s \right]. \tag{13}$$

Now, if Einstein's Eqs. (4) and (5) are valid then the terms in square brackets in Eqs. (12) and (13) should be equal to constant C. This implies that the following two equations

$$M_v I_v - M_s I_s = C, \tag{14}$$

$$(M_v - m_v)I'_v - (M_s - m_s)I'_s = C \tag{15}$$

should hold true. Now if the emission does not affect internal energy per unit mass of the body as viewed in different reference frames S_s and S_v, respectively, then

$$I_s = I'_s, \quad I_v = I'_v. \tag{16}$$

Substituting it into Eq. (14) and subtracting resulting equation from Eq. (15) yield *hidden condition*

$$m_s I'_s = m_v I'_v. \tag{17}$$

This condition suggests that values for internal energy associated with decrease in mass of the body as measured in S_s and S_v, respectively, are identical. *Consequently the hidden condition means that internal energy of any body should have identical value when measured in S_s and S_v and which is perfectly valid within the framework of relativity.* In view of this, then we have

$$M_s I_s = M_v I_v, \quad M_s I'_s = M_v I'_v \tag{18}$$

and substituting it in Eqs. (14) and (15) we obtain

$$C = 0. \tag{19}$$

The obtained value of $C = 0$ suggests that Einstein could have avoided invoking argument (EA) for writing Eqs. (4) and (5) and using C altogether and still could have derived the mass-energy equivalence relation by invoking the above mentioned hidden condition related to internal energy. This means that the new derivation should obtain Eq. (6) from Eq. (3) without using Eqs. (4) and (5) and which is now presented below.

Now consider Eqs. (12) and (13) which were obtained by writing total energy of body E and H as summation of kinetic and internal energies. The obtained hidden condition as described above is a valid condition within the framework of relativity. So using this condition in Eqs. (12) and (13), we obtain *exact* equations

$$H_0 - E_0 = K_0, \tag{20}$$

$$H_1 - E_1 = K_1 \tag{21}$$

which do not include any C and are derived without invoking EA. These Eqs. (20) and (21) further yield

$$(H_0 - E_0) - (H_1 - E_1) = K_0 - K_1 \tag{22}$$

without invoking EA and Einstein's Eqs. (4) and (5), thus making Ives's criticism a void. Then, Eqs. (22) and (3) yield

$$K_0 - K_1 = L[\frac{1}{\sqrt{1 - v^2/c^2}} - 1] \tag{23}$$

which is identical to Eq. (6) and from which mass-energy equivalence relation Eq. (7) follows.

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[2] W. L. Fadner, Did Einstein really discover "$E = mc^2$?", Am. J. Phys. **56**, 114-122 (1988).

[3] J. Stachel and R. Torretti, Einstein's first derivation of mass-energy equivalence, Am. J. Phys. **50**, 760-763 (1982).

[4] H. E. Ives, Derivation of the mass-energy relation, J. Optical Soc. Am. **42**, 540 (1952). Also, reprinted in "The EINSTEIN Myth and the IVES Papers", 182-185, (1979), edited by D. Turner and R. Hazelett, published by The Devin-Adair Company.

[5] A. Einstein, Ann. der Physik **35**, (1911). An English translation "On the Influence of Gravitation on the Propagation of Light" appeared in "The Principle of Relativity", 99-108, (1952), Dover Publications, Inc., translated by W. Perrette and G. B. Jeffery.

6 Ives's criticism[4]: Using the exact expression for kinetic energies K_0 and K_1, Ives showed that

$$(H_0 - E_0) - (H_1 - E_1) = \frac{L}{m_s c^2}(K_0 - K_1)$$

and further he considered it as the difference of two relations (similar to Einstein's argument EA), written as

$$H_0 - E_0 = \frac{L}{m_s c^2}(K_0 + C),$$

$$H_1 - E_1 = \frac{L}{m_s c^2}(K_1 + C).$$

Then he wrote "these are *not*

$$H_0 - E_0 = K_0 + C,$$

$$H_1 - E_1 = K_1 + C.$$

They differ by the multiplying factor $\frac{L}{m_s c^2}$. What Einstein did by setting down these equations (as "clear") was to introduce the relation $\frac{L}{m_s c^2} = 1$. Now this is the very relation the derivation was supposed to yield."